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The Cornell Potential from General Geometries in AdS/QCD

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Abstract

We consider the heavy quark-antiquark potential in the AdS / QCD correspondence, extending recent calculations to a more general family of geometries. We show explicitly that Cornell-like behaviour is a consequence of this general geometry, provided certain conditions are satisfied. As an explicit example, we consider the application of our results to a recently calculated AdS-like metric deformed by back-reaction effects. We find that tuning the long-distance behaviour of the potential leads to a discrepancy at small distances, and discuss how to better constrain AdS / QCD geometries.

1 Introduction

Regardless of whether string theory provides a fundamental description of nature, there is mounting evidence in favour of its being a useful tool for understanding strongly coupled gauge theories. The AdS / CFT correspondence of Maldacena [1] conjectures a mathematical equivalence between the low energy limit of type IIB string theory on $AdS_5 \otimes S^5$ and a conformal field theory, $\mathcal{N} = 4$ $U(N)$ Super-Yang-Mills (SYM) for large N, on the boundary of this space. Since this initial conjecture, various generalisations of the duality principle have been considered involving other gauge theories and their proposed gravity duals. Crucially, a weakly coupled string theory corresponds to a strongly coupled gauge theory, and it is this apparent fact that generates much phenomenological interest in such correspondences. The hope is that a geometry might be found whose dual field theory mimics Quantum Chromodynamics (QCD), thus facilitating the use of string theory to ascertain the properties of strong interactions. There are two main approaches. Firstly, one can develop consistent string theories and try to ascertain the properties of their field theory duals, with the hope of edging closer to QCD. Such theories are ten dimensional and must then be compactified with consequent ambiguities. A second more phenomenological approach is to try to guess a (string-inspired) effective field theory in five-dimensional geometry whose dual (4-dimensional) theory then has QCD-like properties. This is known as the AdS / QCD approach by analogy with Maldacena's conjecture, although the term is slightly misleading due to the fact that deformed Anti-de-Sitter spaces are usually considered. There are then no ambiguities due to choice of compactification manifold, but the choice of 5-dimensional geometry one starts with is itself undetermined. Thus, it is important to classify the properties of various geometries and to examine the constraints which may be imposed to rule out various alternative geometries. One such constraint, that of the behaviour of the heavy quark-antiquark potential, is considered in this paper.

A five-dimensional holographic hadron model was introduced in [2]. A class of models exhibiting linear confinement was examined in [3, 4]. In [5] the heavy quark-antiquark potential was considered in a particular geometry belonging to this class, and found to be consistent with the *Cornell potential* [6]:

$$V(r) = -\frac{\kappa}{r} + \frac{r}{a^2} + C, \tag{1}$$

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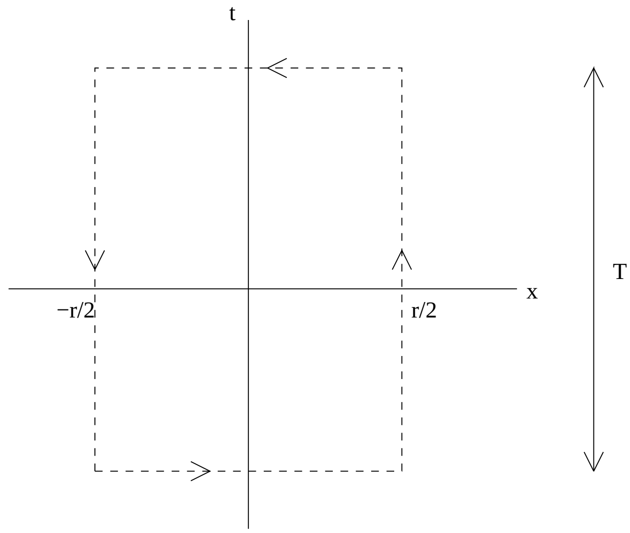

Figure 1: Wilson loop used to derive the interquark potential, where the colour sources are at $x = \pm r/2$ and the limit $T \to \infty$ is taken.

where the coefficients can be found by fitting to Charmonium spectra. The potential can also be measured on the lattice (see [7] for a review). In this paper we generalise the ansatz for the initial geometry, and examine the implications for Cornell-like behaviour. We find that a Cornell-like potential arises quite generally subject to certain conditions on the starting geometry. As a special case, we examine a recently calculated geometry of [8], in which the back-reaction of the 5-dimensional quark condensate fields are used to deform an initial AdS space.

The paper is laid out as follows. In section 2 we recall how to calculate the heavy quark-antiquark potential from a string world-sheet in 5-dimensions spanning a Wilson line on the 4-dimensional boundary [9, 5]. Applying a general ansatz for the AdS / QCD geometry, we show how a Cornell-like potential results. In section 3 we present explicit results for the back-reacted geometries of [8], and in section 4 we discuss our results and conclude.

2 The Heavy Quark-Antiquark Potential

In field theory, the potential between two sources of the gauge field is calculated from the expectation value of a Wilson loop $W(\mathcal{C})$ connecting them in space-time (see figure 1). In fact, one has:

$$< W(\mathcal{C}) > \propto e^{-TV(r)} \tag{2}$$

where the limit $T \to \infty$ is understood and $V(r)$ is the separation-dependent potential. According to the AdS / CFT correspondence, the expectation value of a Wilson loop in the boundary conformal field theory is related to the extremal area of a string world sheet in the higher-dimensional theory, which spans the curve $\mathcal{C}$ [9], as shown in figure 2. One has:

$$< W(\mathcal{C}) > \propto e^{-A}, \tag{3}$$

where A is the area of the string worldsheet. Combining this with equation (2), the potential (up to a constant) is given by:

$$V(r) = \frac{1}{2\pi\alpha' T} \int d^2\xi \sqrt{\det g_{nm} \partial_\alpha X^n \partial_\beta X^m}, \tag{4}$$

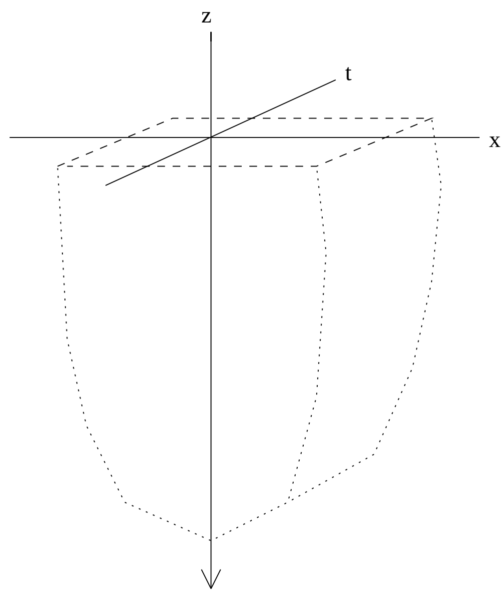

Figure 2: Schematic depiction of the AdS / CFT prescription for obtaining the expectation value of the Wilson line in terms of a string worldsheet [9].

where the Nambu-Goto action for the string worldsheet has been used, and it is understood that this worldsheet has extremal area. Here α' is the string tension, g_{nm} the space-time metric, $X^n(\boldsymbol{\xi})$ are the space-time coordinates and $\boldsymbol{\xi} = (\xi_0, \xi_1)$ are the worldsheet coordinates. One can now find the form of the potential by assuming a given background metric $g_{nm}(X)$, and it is therefore at this point that the model-dependence enters. This method was applied in the framework of the AdS / QCD approach in [5, 10]. Here we assume a general form of the metric:

$$ds^2 = f(z)\, dx^\mu dx_\mu + \frac{dz^2}{z^2}, \tag{5}$$

where z is the fifth dimension and x^μ the 4-dimensional coordinates. If $f(z) = z^{-2}$, this gives a Euclidean AdS space. The possible form of the deformation function $f(z)$ will be considered later. Substituting equation (5) into equation (4) and choosing $\xi_0 = t$ and $\xi_1 = x$ (possible because of reparameterisation invariance of the worldsheet), one obtains after a little algebra:

$$V(r) = \frac{g}{2\pi} \int_{-\frac{r}{2}}^{\frac{r}{2}} dx f(z) \left[1 + \frac{(z')^2}{z^2 f(z)}\right]^{\frac{1}{2}}, \tag{6}$$

where the prime denotes a derivative with respect to x, and $g = 1/\alpha'$. One must extremise this quantity, thus considering the "Lagrangian":

$$L = f(z) \left(1 + \frac{(z')^2}{z^2 f(z)}\right)^{\frac{1}{2}}. \tag{7}$$

Noting the lack of explicit dependence of L on x, there is a first integral:

$$\begin{aligned} H &= z' \frac{\partial L}{\partial z'} - L \\ &= -f(z) \left(1 + \frac{(z')^2}{z^2 f(z)}\right)^{-\frac{1}{2}} = \text{const.} \end{aligned} \tag{8}$$

By symmetry, $z(x)$ has its extremal value at $z = z_0$ at $x = 0$. Evaluating H at both z and z_0 then gives:

$$f_0 \equiv f(z_0) = f(z) \left(1 + \frac{(z')^2}{z^2 f(z)}\right)^{-\frac{1}{2}}, \tag{9}$$

from which one finds:

$$z' = z\sqrt{f(z)} \left[\left(\frac{f(z)}{f_0}\right)^2 - 1\right]^{\frac{1}{2}}. \tag{10}$$

Naïvely, the radius and potential are then given parametrically in terms of z_0 by:

$$r(z_0) = \int_{-\frac{r}{2}}^{\frac{r}{2}} dx = 2\int_0^{z_0} \frac{dz}{z\sqrt{f(z)}} \left[\left(\frac{f(z)}{f_0}\right)^2 - 1\right]^{-\frac{1}{2}} \tag{11}$$

and:

$$V(z_0) = \frac{g}{\pi} \int_0^{z_0} dz \frac{\sqrt{f}}{z} \left[1 - \left(\frac{f_0}{f(z)}\right)^2\right]^{-\frac{1}{2}}, \tag{12}$$

where we have used equation (10) to change the integration variable from x to z. In fact, the potential given by equation (12) diverges and must be regularised [9]. We can see this here as follows. It was shown in [3] that $f \sim 1/z^2$ as $z \to 0$ (in the absence of a dilaton) is a (not sufficient) condition for linear confinement. Furthermore, the back-reacted geometries of [8] also imply this behaviour, which is nothing more than the condition that the geometry (5) be asymptotically AdS. Assuming this behaviour for the deformation function, one can replace the lower limit of the integral in equation (12) by $z = \delta$ to obtain:

$$V(z_0, \delta) = \frac{g}{\pi}\frac{1}{\delta} + V_R(z_0, \delta), \tag{13}$$

where $V_R(z_0, \delta)$ is the regularised energy[2]:

$$V_R = \frac{g}{\pi} \left\{ -\frac{1}{z_0} + \int_\delta^{z_0} dz \left[\frac{\sqrt{f(z)}}{z} \left[1 - \left(\frac{f_0}{f(z)}\right)^2\right]^{-\frac{1}{2}} - \frac{1}{z^2} \right] \right\}, \tag{14}$$

in which one may take $\delta \to 0$. It is not possible to eliminate the parameter z_0 to obtain $V(r)$ directly. In general, one must instead calculate $V(r)$ numerically by finding $r(z_0)$ and $E(z_0)$ by numerical integration for a series of values of z_0. To aid numerical convergence of these integrals, it is useful to introduce $\tilde{f}(z) = z^2 f(z)$ so that equations (11, 14) become:

$$r(z_0) = 2\int_0^{z_0} dz \frac{1}{\sqrt{\tilde{f}}} \left[\frac{z_0^4 \tilde{f}^2}{z^4 \tilde{f}_0^2} - 1\right]^{-\frac{1}{2}} \tag{15}$$

and:

$$V_R(z_0) = \frac{g}{\pi} \left\{ -\frac{1}{z_0} + \int_0^{z_0} \frac{dz}{z^2} \left[\sqrt{\tilde{f}} \left(1 - \frac{z^4 \tilde{f}_0^2}{z_0^4 \tilde{f}^2}\right)^{-\frac{1}{2}} - 1 \right] \right\}. \tag{16}$$

Asymptotic AdS behaviour corresponds to $\tilde{f}(z) \to 1$ as $z \to 0$.

[2] Note a similar expression for a different starting geometry has been given in [5].

Before pursuing further a numerical evaluation of the potential, let us first examine the asymptotic behaviour of $V(r)$ at small and large r. For small r the calculation is analogous to that in [5]. One can expand equation (15) about $z_0 = 0$ to obtain:

$$r(z_0) \simeq 2\int_0^{z_0} dz \left(\frac{z_0^4}{z^4} - 1\right)^{-\frac{1}{2}}$$
$$= 2z_0 \int_0^1 dv\, v^2 (1 - v^4)^{-\frac{1}{2}}, \tag{17}$$

where $v = z/z_0$. Evaluating the integral one finds:

$$r \simeq \frac{z_0}{\rho}, \quad \rho = \frac{\Gamma^2(1/4)}{(2\pi)^{3/2}}. \tag{18}$$

Thus, small values of z_0 correspond to small values of $r(z_0)$. One may similarly expand the expression for $V_R(z_0)$ in this limit to obtain:

$$V_R(z_0) \simeq \frac{g}{\pi}\left\{-\frac{1}{z_0} + \int_0^{z_0} \frac{dz}{z^2}\left[\left(1 - \frac{z^4}{z_0^4}\right)^{-\frac{1}{2}} - 1\right]\right\}, \tag{19}$$

which from equation (18) gives:

$$V_R(r) \simeq -\frac{g}{2\pi\rho^2 r}, \quad r \to 0, \tag{20}$$

which is the Cornell-like behaviour of equation (1). That the same behaviour was noted in [5] is no surprise, as this argument explicitly shows that the $r \to 0$ behaviour of the potential is independent of $f(z)$ for an asymptotically AdS starting geometry. Note that g is the only free parameter influencing $V(r)$ at small r.

We now examine the large r behaviour of the potential. From equation (15) one sees that $r(z_0)$ diverges at finite z_0 given by the criterion that:

$$\frac{z_0^4 \tilde{f}^2}{z^4 \tilde{f}_0^2} - 1 = 0 \tag{21}$$

has a solution for $z < z_0$. This will occur at a value $z_0 = z_0^*$ given by:

$$\frac{d}{dz}\left[z_0^4 \tilde{f}^2\right]_{z_0^*} = \frac{d}{dz}\left[z^4 \tilde{f}_0^2\right]_{z_0^*}, \tag{22}$$

which gives the equation for z_0^*:

$$z_0^* \left.\frac{d\tilde{f}}{dz}\right|_{z_0^*} = 2\tilde{f}_0^*, \tag{23}$$

where $\tilde{f}_0^* = \tilde{f}(z_0)|_{z_0^*}$. In general one is only able to solve this equation numerically for z_0^*, although we will see in the next section that an analytical solution is possible for the back-reacted geometry of [8]. Let us derive the behaviour of $r(z_0)$ and $V_R(z_0)$ near $z_0 = z_0^*$, which are needed to find the large r behaviour of the potential. One may expand the integrand of equation (15) in $\varepsilon = z_0^* - z$ to give:

$$r(z_0) \simeq 2\int_\epsilon^{z_0} d\varepsilon \left\{\frac{z_0^*}{\sqrt{\tilde{f}_0^*}}\left[\frac{z_0^*}{2}\frac{(\tilde{f}_0^*)''}{(\tilde{f}_0^*)^2} - 2\right]^{-\frac{1}{2}} \frac{1}{\varepsilon} + \mathcal{O}(\epsilon^0)\right\}, \tag{24}$$

where now primes denote differentiation with respect to z, and ϵ is a regularising parameter. Note we have used the condition (23) in obtaining this equation. Changing the integration variable from z to ε one has:

$$r(\epsilon) = -\frac{2z_0^*}{\sqrt{\tilde{f}_0^*}}\left[\frac{(z_0^*)^2}{2}\frac{(\tilde{f}_0^*)''}{\tilde{f}_0^2} - 2\right]^{-\frac{1}{2}} \log\epsilon + \mathcal{O}(\epsilon^0), \quad \epsilon \to 0. \tag{25}$$

Similarly, one finds from equation (16) in the same limit:

$$V_R(z_0) \simeq \int_{\epsilon}^{z_0} d\varepsilon \left\{ \frac{g\sqrt{\tilde{f}_0^*}}{\pi z_0^*} \left[\frac{(z_0^*)^2}{2} \frac{(\tilde{f}_0^*)''}{\tilde{f}_0^2} - 2 \right]^{-\frac{1}{2}} \frac{1}{\varepsilon} + \mathcal{O}(\varepsilon^0) \right\} \tag{26}$$

from which one finds:

$$V_R(\epsilon) = -\frac{g\sqrt{\tilde{f}_0^*}}{\pi z_0^*} \left[\frac{(z_0^*)^2}{2} \frac{(\tilde{f}_0^*)''}{\tilde{f}_0^2} - 2 \right]^{-\frac{1}{2}} \log \epsilon + \mathcal{O}(\epsilon^0), \quad \epsilon \to 0. \tag{27}$$

Putting equations (25, 27) together, one has:

$$V_R(r) \to \frac{g\tilde{f}_0^*}{2\pi (z_0^*)^2} r, \quad r \to \infty, \tag{28}$$

which is a linear potential energy at large distances and thus again a Cornell-like behaviour. In fact, one could have concluded this directly from equations (15, 16) without explicitly calculating the integrals. The integral for $V_R(z_0)$ contains the same divergence as $r(z_0)$, and it is this fact that leads to a proportionality between them when $r(z_0)$ diverges. However, the prefactor in square brackets in equations (25, 28) is important. Clearly the above argument breaks down if:

$$(\tilde{f}_0^*)'' < \frac{4(\tilde{f}_0^*)^2}{(z_0^*)^2}, \tag{29}$$

as then r, $V_R \notin \mathbb{R}$. Using the condition (22) we can rewrite this as:

$$(\tilde{f}_0^*)'' < (\tilde{f}_0^*)'^{\,2}. \tag{30}$$

One can express this in a different way in order to compare with the literature by making the following "warp factor" ansatz for the deformation function:

$$f(z) = e^{2A(z)}. \tag{31}$$

Such a form has been assumed directly in e.g. [3, 8], where $A(z) \sim -\log(z)$ to give the AdS behaviour of the geometry as $z \to 0$. It is also used in a modified form in [5]. The condition (30) for reality of the interquark separation and potential energy becomes:

$$\bar{A}''(z_0^*) < 2[\bar{A}'(z_0^*)]^2 (e^{2\bar{A}(z_0^*)} - 1) \quad \Rightarrow r,\, V_R \notin \mathbb{R}, \tag{32}$$

where $\bar{A}(z)$ denotes the function $A(z)$ with the $\log z$ term subtracted. Equation (30) carries the simple physical interpretation of a certain positive curvature of the warp factor at the value of z_0 at which the quarks are infinitely separated (illustrated in figure 3). This condition is consistent with back-reacted geometries of [8], the geometry assumed in [5], and the conditions for linear confinement found in [3][3].

A stronger condition on the geometry arises from considering equations (15, 16) at general z. Requiring positivity of the factor in square brackets in equation (15) yields:

$$\frac{\tilde{f}}{z^2} < \frac{\tilde{f}_0}{z_0^2}, \quad r \notin \mathbb{R}. \tag{33}$$

We will examine this condition in the specific case of the back-reacted geometries of [8] in the next section.

[3] That is, once subtleties involved in the Wick rotation to Euclidean space have been accounted for. See the following section and appendix A.

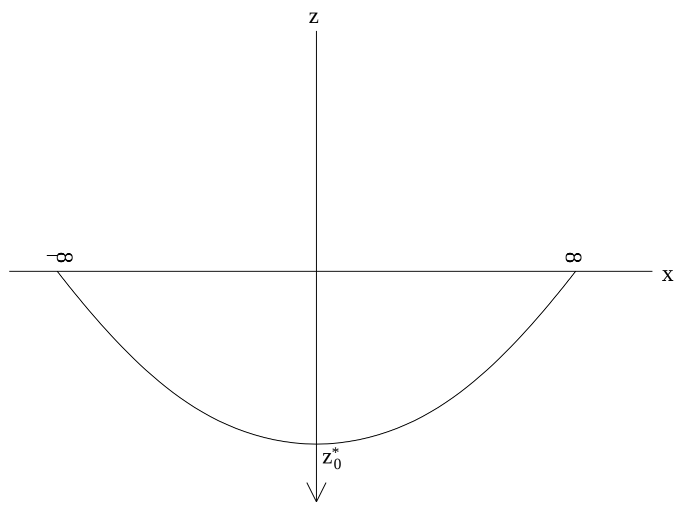

Figure 3: Illustration of the value $z_0 = z_0^*$ at which the interquark separation diverges.

3 Results in a Back Reacted Geometry

The holographic hadron model of [2] features a five dimensional field dual to the 4-dimensional bilinear quark operator $q\bar{q}$ permeating a fixed 5-dimensional (AdS) geometry. This is an approximation which ignores the fact that the action for the 5-dimensional field theory in a curved space-time couples the quark field to the metric. Thus, the presence of a bulk field will itself deform the AdS geometry, leading to the deformation factor $f(z)$ discussed above. This effect was taken into account in [8], where a differential equation is given for the "warp factor" $A(z)$ of equation (31) in terms of the bulk scalar field [4]. One solves this equation subject to a suitable ansatz for this field, and more than one example is given in [8]. We use as our example the following back-reacted warp-factor (denoted "Model 1" in [8]):

$$A(z) = -\log z + \frac{m_q^2}{48} z^2 - \frac{m_q \sigma}{32} z^4 + \frac{\sigma^2}{48} z^6, \tag{34}$$

coming from the following ansatz [2] for the bulk field X dual to the bilinear quark operator[5]:

$$X = \frac{m_q}{2} z + \frac{\sigma}{2} z^3. \tag{35}$$

Care is needed to obtain equation (34) from the result given in [8] due to the use of a Euclidean signature here. This technicality is explained in appendix A. Using equations (23, 31), we can find the value $z_0 = z_0^*$ at which the interquark separation diverges. It is given by the equation:

$$\frac{\sigma^2}{8}(z_0^*)^6 - \frac{m_q \sigma}{8}(z_0^*)^4 + \frac{m_q^2}{24}(z_0^*)^2 - 1 = 0, \tag{36}$$

which is a cubic equation for $(z_0^*)^2$, whose real solution for general m_q and σ is:

$$z_0^*(m_q, \sigma) = \left[\frac{m_q \sigma + \sqrt[3]{216\sigma^4 - m_q^3 \sigma^3}}{3\sigma^2} \right]^{\frac{1}{2}}. \tag{37}$$

[4] Note that the dilaton ϕ is assumed constant in this analysis, consistent with the semi-classical approximation.
[5] This is not to be confused with the string space-time coordinates introduced earlier.

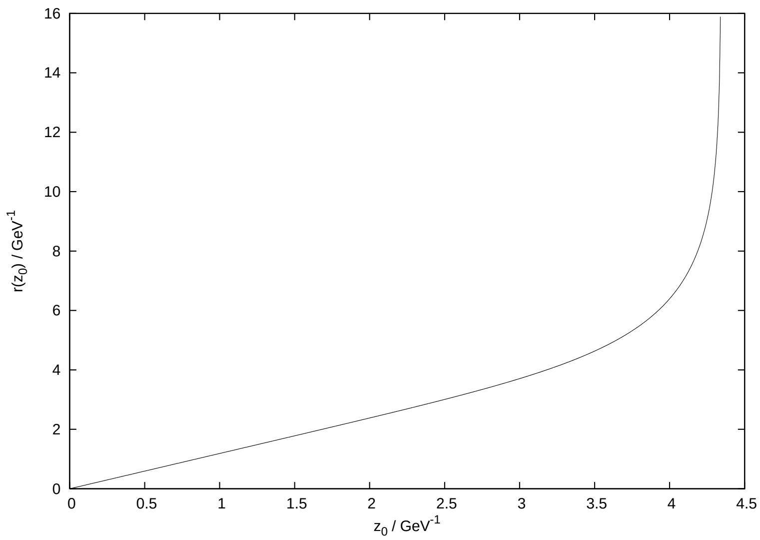

Figure 4: Behaviour of $r(z_0)$ for $z_0 \leq z_0^*$, as given by the geometry of equation (34) with parameters (38).

In [8] the values of m_q and σ were constrained by considering the number of quark colours, meson masses and the pion decay constant, thus leading to the values:

$$m_q = 2.4\text{MeV}, \quad \sigma = (326\text{MeV})^3. \tag{38}$$

Then one has:

$$z_0^* = 4.341\,\text{GeV}^{-1}, \quad \tilde{f}_0^* = 1.395 \tag{39}$$

for the extremal values of the parameter z_0 and the deformation function. One can then verify for these values that the geometry satisfies the conditions (32, 33) needed for the Cornell behaviour. A plot of $r(z_0)$ is shown in figure 4, and verifies the divergence at z_0^* given by equation (39). From equation (28), one has for these values of m_q and σ:

$$V_R \to (0.01178g)r, \quad r \to \infty \tag{40}$$

where g has yet to be fixed. In [5], this parameter is fixed from the Cornell potential, taking $a = 2.34\text{GeV}^{-1}$ in equation (1). Adopting this procedure here, one finds $g = 15.50$. This is much larger than the value $g = 0.94$ given in [5] (albeit for a slightly different geometry), this difference mostly arising from the parameters of the warp factor in the geometry. If one fixes the values of m_q and σ from [8], one has no choice but to fix g in this way in order to obtain the correct long-distance behaviour. The resulting potential is compared with the Cornell potential in figure 5. The constant term of the AdS potential is adjusted so that equality is reached with the Cornell potential at asymptotically large distances, in order to more easily facilitate a comparison. One sees a marked deviation between the two potentials away from the asymptotically long-distance regime. Indeed the value of g obtained gives, via equation (20), the short-distance behaviour:

$$V_R(r) \to -\frac{3.541}{r}, \quad r \to 0 \tag{41}$$

in marked contrast to the Cornell potential taken from [5], which has $\kappa = 0.48$ in equation (1). This difference has arisen from fixing m_q and σ according to [8], then using the long-distance behaviour of the Cornell potential to fix the only remaining free parameter g. There are then no free parameters left which influence the short-distance behaviour - as noted in section 2, g is the only parameter governing this regime. Although one does not expect the semi-classical theory used here to give a correct description at short

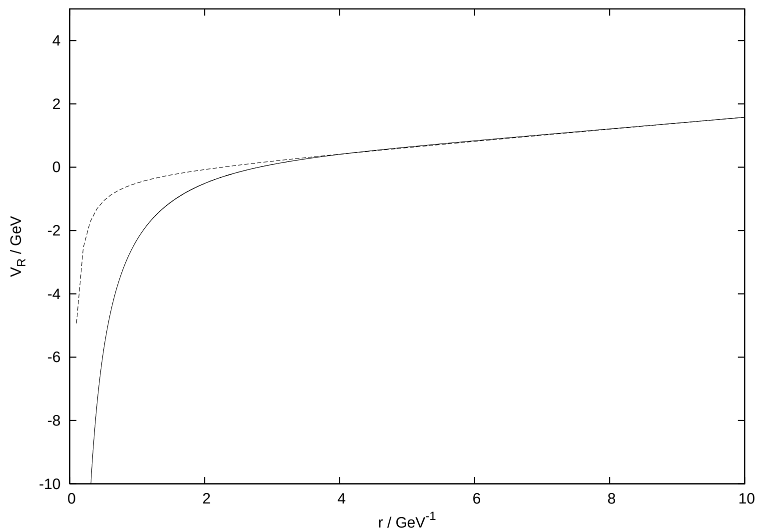

Figure 5: The potential $V_R(r)$ arising from the geometry (34) with parameters as discussed in the text (solid). Also shown is the Cornell potential quoted in [5] (dashed). The constant term in the AdS result has been adjusted.

distances, the possibility exists of using the parameters in the deformation function $f(z)$ to tune the long-distance behaviour of the potential. Then g can be used so as to extend agreement between the AdS / QCD and Cornell potentials into the moderate r region, and the parameters m_q and σ varied to tune the long-distance behaviour. There are more parameters in the deformation function, however, than are needed to fix the long-distance potential[6]. Furthermore, it is inappropriate to vary the geometrical parameters in order to tune the potential energy at the expense of other observables (such as those used to fix m_q and σ in [8]) which depend on them. A combined analysis must be performed using all calculable observables, with the possible addition of further parameters to the warp factor of equation (34). This deserves further study.

4 Discussion

In this paper we have extended the calculation of the Cornell potential within the AdS / QCD approach given in [5] to encompass a more general family of geometries. We used the deformed Euclidean AdS metric given by equation (5), as this is the ansatz used (after Wick rotation) by [8] in their analysis of back-reaction effects. The arguments given here are clearly applicable to the type of metric used in [5].

We find that a Cornell-like potential is a consequence of any geometry that satisfies the conditions (33) and more specifically (32), where the latter condition is interpreted as a (minimum) positive curvature of the warp factor at the extremal value of z_0 at which the interquark separation diverges.

In the specific case of the “Model I” back-reacted geometry of [8], we provide an example potential for the choice of parameters given in that paper. Once these are fixed, one can fix the only remaining parameter g by matching to the long-distance behaviour of the Cornell potential. This gives a marked deviation between the AdS and Cornell potentials away from this regime, with a coefficient of the r^{-1} behaviour at short distances that shows little agreement with the Cornell result. It thus seems more sensible to allow g to

[6]One could also try to reproduce the constant term in the Cornell potential. However, a constant term in the potential energy makes no difference to the dynamics.

vary along with the parameters of the geometry in order to extend agreement between the AdS / QCD and Cornell potentials into the moderate r regime. This must be done in a way that is consistent with other observables calculated from the AdS / QCD approach, and thus a clear program for constraining possible geometries in a phenomenological fashion presents itself. One could argue that the stringy approach is not expected to work well at short distances, where the semi-classical approximation breaks down. Nevertheless, what quantitatively constitutes a short distance is unclear and it would be interesting to see whether by considering more observables and parameters, a consistent and concise geometry in better agreement with all available lattice and experimental data might be found.

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A The Back-Reacted Geometry in Euclidean Space

In [8] back reaction effects are considered starting from the geometry:

$$ds^2 = e^{-2\tilde{A}(y)} dx_\mu dx^\mu - dy^2, \tag{42}$$

where we denote the warp factor with a tilde to avoid confusion with $A(z)$ as defined in this paper. Introducing $z = e^y$, one then has:

$$ds^2 = e^{-2\tilde{A}(z)} dx_\mu dx^\mu - \frac{dz^2}{z^2}. \tag{43}$$

Note that instead of using a different symbol for the warp factor in terms of z, we clarify any ambiguity by denoting the arguments of this function explicitly. To obtain a Euclidean signature one can Wick rotate:

$$dx^\mu \to \imath dx^\mu, \quad z \to \imath z \tag{44}$$

to obtain:

$$ds^2 = \left[e^{-2\tilde{A}(\imath z)} \left(dt^2 + d\boldsymbol{x}^2 \right) - \frac{dz^2}{z^2} \right]. \tag{45}$$

The "Model I" warp factor of [8] is given in Minkowski space by:

$$\tilde{A}(z) = \log(z) + \frac{m_q^2}{48} z^2 + \frac{m_q \sigma}{32} z^4 + \frac{\sigma^2}{48} z^6. \tag{46}$$

Substituting this in equation (45) gives:

$$|ds^2| = \left[\frac{1}{z^2} \exp\left(\frac{m_q^2}{24} z^2 - \frac{m_q \sigma}{16} z^4 + \frac{\sigma^2}{24} z^6 \right) (dt^2 + d\boldsymbol{x}^2) + \frac{dz^2}{z^2} \right]. \tag{47}$$

We note that the requirement of a positive sign in a warp factor consisting only of $A(z) \sim z^2$ in Euclidean space was noted already in [4] based on the need for a discrete spectrum of meson masses.

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